Name:

Spring 2010 Math 245 Final Exam

Please read the following directions:
Please write legibly, with plenty of white space. Please put your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. You may NOT use your book, notes, calculators or other aids. This exam will last 120 minutes; pace yourself accordingly. If you are done early, you may leave – but NOT during the last five minutes of the exam, during which you are asked to remain quiet and in your seat. Good luck!

Problem	Min Score	Your Score	Max Score
1.	5		10
2.	5		10
3.	5		10
4.	5		10
5.	5		10
6.	5		10
7.	5		10
8.	5		10
9.	5		10
10.	5		10
11.	5		10
12.	5		10
13.	5		10
14.	5		10
15.	5		10
16.	5		10
17.	5		10
18.	5		10
19.	5		10
20.	5		10
Total:	100		200

Problem 1. Carefully define the following terms:

a. proposition

b. contradiction

c. counterexample

Problem 2. Carefully define the following terms:

a. converse

b. contrapositive

c. modus ponens

Problem 3. Carefully define the following terms:

a. subset

b. power set

c. absolute complement

Problem 4. Carefully define the following terms:

a. partial order

b. total order

c. equivalence relation

Problem 5. Carefully define the following terms:

a. ceiling

b. bijection

c. function

Problem 6. Carefully define the following terms:

a. event

b. O notation

c. pigeonhole principle

Problem 7. Carefully define the following terms:

a. path

b. Eulerian path

c. Hamiltonian path

Problem 8. Carefully define the following terms:

a. ordinal

b. cardinal

c. countable

Problem 9. How many ways can a hand of 2 cards be drawn from an ordinary 52 card deck?

Problem 10. How many such hands form a pair? (i.e. they are the same rank, but of different suits)

Problem 11. What is the probability of such a hand being a pair?

Problem 12. If you lose \$1 for a non-pair, and win \$5 for a pair, what is your expected profit?

Problem 13. Prove that if $a|b$ and $b|c$, then $a|c$.

Problem 14. Prove that $(p \to q) \vee r \equiv (p \to r) \vee q$.

Problem 15. Prove that the product of two odd numbers is odd.

Problem 16. Prove that $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.

Problem 17. Prove that $n^2 + 1 \in O(n^2)$.

Problem 18. Prove that $n^3 + 1 \notin O(n^2)$.

Problem 19. Draw (and label) a simple graph that has a Hamiltonian circuit but no Eulerian path or circuit.

Problem 20. Solve the recurrence given by $a_0 = a_1 = 1, a_n = 3a_{n-1} - 2a_{n-2}$ $(n \geq 2)$.